

Flavor-singlet baryons in the graded symmetry approach to partially quenched QCD

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Progress in the calculation of the electromagnetic properties of baryon excitations in lattice QCD presents new challenges in the determination of sea-quark loop contributions to matrix elements. A reliable estimation of the sea-quark loop contributions represents a pressing issue in the accurate comparison of lattice QCD results with experiment. In this article, an extension of the graded symmetry approach to partially quenched QCD is presented, which builds on previous theory by explicitly including flavor-singlet baryons in its construction. The formalism takes into account the interactions among both octet and singlet baryons, octet mesons, and their ghost counterparts; the latter enables the isolation of the quark-flow disconnected sea-quark loop contributions. The introduction of flavor-singlet states enables systematic studies of the internal structure of Λ -baryon excitations in lattice QCD, including the topical $\Lambda(1405)$.

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I. INTRODUCTION

A. Quark-loop contributions to matrix elements

The calculation of hadronic matrix elements in nonperturbative QCD typically encounters non-trivial contributions from quark-flow disconnected diagrams in which the current interacts with the hadrons of interest, first through a virtual sea-quark loop. An example process for a baryon undergoing a virtual meson-baryon transition, $B \rightarrow B\Phi$, involving a disconnected sea-quark loop, is depicted in Fig. 1. These disconnected-loop diagrams are difficult to calculate in numerical simulations of QCD due to the space-time coordinate dependence of both the source and sink of the quark propagator at the current insertion point. As the loop is correlated with other quark degrees of freedom only via gluon exchange, resolving a nontrivial signal requires high statistics through innovative methods.

While there has been recent success in isolating disconnected-loop contributions in ground-state baryon matrix elements, even at non-zero momenta [1, 2], the challenging nature of these calculations suggests that resolving similar contributions for hadronic excitations in lattice QCD [3–16] will be elusive.

However, it is here that the meson-baryon dressings of hadron excitations are particularly important. Consider, for example, the $\Lambda(1405)$ resonance which is of particular contemporary interest. It is widely agreed that there is a two-pole structure for this resonance [17–28], associated with attractive interactions in the $\pi\Sigma$ and $\bar{K}N$ channels. The two poles lie near the $\pi\Sigma$ and $\bar{K}N$ thresholds and both channels can contribute to the flavour singlet components of the resonance, studied in detail herein. This two-pole structure, as first analyzed in Ref. [17], has now been entered into the Particle Data Group tables [29]. This achievement reflects decades of research using the successful chiral unitary approaches [30–42], which have elucidated the two-pole structure [17]. In describing the $\Lambda(1405)$ resonance at quark masses larger than the physical masses, the introduction of a bare state dressed by meson-baryon interactions has also been explored [28]. In

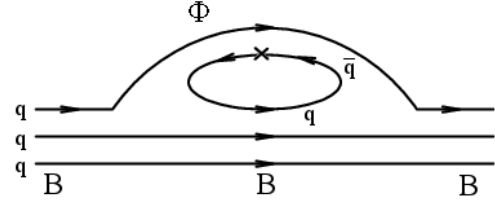


FIG. 1. The quark-flow disconnected diagram for the process $B \rightarrow B\Phi$. The cross on the disconnected quark-loop line indicates the insertion of the matrix-element current. For an incoming baryon, B , each of the three valence quarks entering on the left hand side may contribute to the internal meson state, Φ . In the graded symmetry approach, the conventional u , d and s sea-quark loops are complemented by their commuting ghost counterparts, \tilde{u} , \tilde{d} and \tilde{s} .

all cases, the disconnected loops of Fig. 1 will make important contributions to the matrix elements of these states.

Recent lattice QCD calculations of the electromagnetic form factors of the $\Lambda(1405)$ have reported evidence of an important $\bar{K}N$ molecular component in the $\Lambda(1405)$ [13]. On the lattice, the strange quark contribution to the magnetic form factor of the $\Lambda(1405)$ approaches zero, signaling the formation of a spin-zero kaon in a relative S -wave about the nucleon. However, the light u and d valence-quark sector of the $\Lambda(1405)$ makes a non-trivial contribution, which can be related to the proton and neutron electromagnetic form factors.

To understand the physics behind this contribution and enable a comparison with experiment, one needs to understand the role of disconnected sea-quark loop contributions to the matrix elements of the $\Lambda(1405)$. These contributions have not been included in present-day lattice QCD calculations and present a formidable challenge to the lattice QCD community.

When disconnected sea-quark loop-contributions are unknown, one often seeks observables insensitive to this contribution. For example, the isovector baryon combination is insensitive to a sea-quark contribution as it is similar for both isospin variants, and cancels. For baryon states that are not in an isovector formation, a reliable estimation of the sea-quark loop contributions remains a pressing issue in the accurate

comparison of lattice results with experiment.

B. Flavor-singlet symmetry

In lattice QCD calculations, the $\Lambda(1405)$ is excited from the vacuum using interpolating fields dominated by a flavor singlet operator. Thus, a complete understanding of the $\Lambda(1405)$ necessarily involves understanding the role of disconnected sea-quark loop contributions in the flavor-singlet sector of effective field theory. Away from the $SU(3)$ -symmetric limit, flavor-singlet and flavor-octet interpolators mix in isolating the $\Lambda(1405)$ [4] and therefore this new information in the flavor-singlet sector will complement the flavor-octet sector.

Flavor symmetry is important in effective field theory as it provides guidance on the meson-baryon couplings to be used in coupled-channel analyses. It also has the ability to describe the relative strength between quark-flow connected and disconnected contributions to observables. This separation is well understood in the flavor-octet sector of effective field theory, but is unknown in the flavor-singlet sector. This gap in understanding is addressed in this article.

It is perhaps important to note that the physical $\Lambda(1405)$ is a mix of flavor symmetries due to the explicit breaking of $SU(3)$ symmetry by the quark masses. Our task is not to determine this mixing, but rather to disclose the separation of connected and disconnected contributions in the flavor-singlet sector for the first time. Thus, in referring to flavor-singlet baryons in the following, it is to be understood that we are referring to the flavor-singlet component of a baryon. In this manner, our results are applicable to Λ -baryons in general.

C. Graded symmetry approach

The separation of connected and disconnected contributions to the baryon couplings can be handled using the graded symmetry approach [43–49] or the diagrammatic approach [50, 51]. Although it has been demonstrated that both methods are consistent with each other [50, 51], in this work, the graded symmetry approach is used because of its ability to be easily generalized to include flavor-singlet baryons. The diagrammatic approach becomes cumbersome when the valence content of the baryon comprises three unique quark flavors (uds) [50].

Once the coupling strength of disconnected sea-quark loop contributions to hadron two-point functions is understood, one can use this information to estimate the size of sea-quark loop contributions to matrix elements [51–57].

The formalism presented herein is based on the original work of Labrenz and Sharpe [43], and builds upon the constructions of that work in a consistent manner. We create a new, completely symmetric baryon field in the graded symmetry Lagrangian, and couple this field to octet mesons and octet baryons. Through the admission of the commuting ‘ghost’ counterparts, introduced to isolate the disconnected loop component of each coupling, the particles collectively fall into representations of the vector subgroup of the graded symmetry

group $SU(3|3)_V$, as in Ref. [43]. The anticommuting behavior of the standard quark degrees of freedom enable our symmetric baryon field in the graded symmetry Lagrangian to represent the flavor-singlet baryon.

Our aim here is to disentangle quark-flow connected and disconnected couplings between singlet baryons and octet mesons and baryons in modern 2 + 1-flavor lattice QCD simulations. As such, there are no issues with the artifacts of quenched QCD, such as a light double-pole η' meson. Moreover, we will consider quark masses similar to those of nature, with $m_u \simeq m_d \leq m_s$.

D. Outline

The structure of the paper is as follows. Section II expands upon the graded symmetry approach to include the couplings of flavor-singlet baryons to octet mesons and baryons, and the Lagrangian is established. In Section III, particle labels for baryons containing a single ghost quark are explicitly written. This partly follows the method described in Ref. [46], with a few differences in the notation, which are introduced for convenience in identifying the quantum numbers and symmetry properties of the baryons considered. As a relevant test case, the flavor-singlet component of the $\Lambda(1405)$ is described in Sec. IV.

II. GRADED SYMMETRY FORMALISM

In this section, the details of the graded symmetry approach are briefly reviewed. In order to derive couplings to flavor-singlet baryons, the standard graded symmetry approach requires a complementary augmentation in order to include interactions between octet and singlet baryons, octet mesons, and their ghost counterparts.

For 2+1 flavor dynamical-fermion lattice QCD calculations, the relevant graded group is $SU(6|3)_V$, composed of three valence-quark flavors, three sea-quark flavors and three ghost-quark flavors [46]. While the sea quark masses are free to differ from the valence sector, the ghost sector masses are matched to the valence sector such that their fermion-determinant contributions exactly cancel the fermion-determinant contributions of the valence sector, thus ensuring the valence sector is truly valence only.

However, a common approach used in practice, when separating connected and disconnected contributions, is to work within the simpler graded group $SU(3|3)_V$ of quenched QCD. Here the standard quark sector is complemented by a ghost quark sector with matching quark masses to render the quark sector to a valence quark sector. Identification of the ghost sector contributions to quark-flow diagrams is then sufficient to identify the sea-quark loop contribution of the standard quark sector that is being removed. This approach is used herein.

We note, however, that there are important differences between the two graded groups, particularly for the flavor-singlet η' meson. In $SU(3|3)_V$ of quenched QCD, the η' appears

only as a double-hairpin diagram, whereas in $SU(6|3)_V$ the η' appears in a partially-quenched generalization of the standard η' . In our case of interest, it is sufficient to hold the masses of each each flavor in the valence, sea and ghost sectors equal, such that all mesons and baryons participating in the diagrams take their standard properties.

In deriving a Lagrangian that couples flavor-singlet baryons to octet baryons and mesons under the graded symmetry subgroup $SU(3|3)_V$, the constructions introduced in Ref. [43] will be used as much as possible, so that the extension to singlet couplings can draw on the previously established formalism.

In the graded symmetry approach, the quark fields are extended to include bosonic ghost counterparts for each flavor. The new field, $Q = (u, d, s, \tilde{u}, \tilde{d}, \tilde{s})^T$, transforms under the vector subgroup of the graded symmetry group $SU(3|3)_V$ as a fundamental representation, analogous to three-flavor chiral perturbation theory (χ PT)

$$Q_i \rightarrow U_{ij} Q_j, \quad \bar{Q}_i \rightarrow \bar{Q}_j U_{ji}^\dagger, \quad (1)$$

where the indices i, j run from 1 to 6.

A. Mesons

The meson field is expanded to a 6×6 matrix, with four blocks of mesons corresponding to the ordinary mesons, $\pi(q\bar{q})$, the entirely-ghost mesons, $\tilde{\pi}(\tilde{q}\tilde{\bar{q}})$, and the fermionic composite mesons $\chi(q\bar{q})$ and $\chi^\dagger(q\bar{q})$ in the off-diagonal blocks

$$\Phi = \begin{bmatrix} \pi & \chi^\dagger \\ \chi & \tilde{\pi} \end{bmatrix}. \quad (2)$$

The elements of the ordinary meson block follow the standard convention

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} + \frac{\eta'}{\sqrt{6}} \mathbb{I}_3. \quad (3)$$

With the inclusion of ghost quarks, the flavor of the ghost quark or antiquark may specified through either the usual quark mixing, to form $\tilde{\pi}^0$, $\tilde{\eta}$ and $\tilde{\eta}'$ particles, or through neutral meson labels $\tilde{\eta}_u$, $\tilde{\eta}_d$ and $\tilde{\eta}_s$. Both representations have their merit. When the full flavor symmetry of the meson dressing is present, the standard representation with $\tilde{\pi}^0$, $\tilde{\eta}$ and $\tilde{\eta}'$ is of particular utility as the mass of the intermediate meson is manifest. Therefore, we consider the following form for χ^\dagger

$$\chi^\dagger = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\pi}^0 + \frac{1}{\sqrt{6}}\tilde{\eta} & \tilde{\pi}^+ & \tilde{K}^+ \\ \tilde{\pi}^- & -\frac{1}{\sqrt{2}}\tilde{\pi}^0 + \frac{1}{\sqrt{6}}\tilde{\eta} & \tilde{K}^0 \\ \tilde{K}^- & \bar{\tilde{K}}^0 & -\frac{2}{\sqrt{6}}\tilde{\eta} \end{pmatrix} + \frac{\tilde{\eta}'}{\sqrt{6}} \mathbb{I}_3. \quad (4)$$

On the other hand, the flavor symmetry can be incomplete. In this case, it is more transparent to work with the neutral $\tilde{\eta}_u$,

$\tilde{\eta}_d$ and $\tilde{\eta}_s$ mesons along the diagonal of Eq. (4). This is useful when considering the ghost sector of a flavor-octet baryon transition to a flavor-singlet baryon and an octet meson.

The group transformation properties of the meson field can be made compatible with the baryon fields via the methods of χ PT, where the exponential of the mesons is incorporated into an axial-vector field

$$\xi(x) = \exp(i\Phi(x)/f_\pi), \quad (5)$$

$$A_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi). \quad (6)$$

The elements of A^μ transform under the graded-symmetry subgroup $SU(3|3)_V$ as

$$A_{ij}^\mu \rightarrow U_{ii'} A_{i'j'}^\mu U_{j'j}^\dagger. \quad (7)$$

B. Octet and singlet baryons

In the graded symmetry approach, the baryon field is generalized to a rank-3 tensor, allowing the three constituent flavors of each baryon to be specified separately by each flavor index. This is convenient, as it readily allows one to identify the valence quark content of a baryon simply by inspection. The introduction of graded symmetry into the transformation matrices, U , in Eq. (1) requires the introduction of a grading factor, defined as

$$\eta_i = \begin{cases} 1 & \text{if } i = 1-3 \\ 0 & \text{if } i = 4-6 \end{cases}, \quad (8)$$

which takes into account the bosonic nature of the ghost quarks – swapping two quark fields incurs a minus sign only if both are anticommuting (Dirac-spinor field) quarks.

The transformation properties of the octet baryon field can be derived from the standard nucleon interpolating field

$$\mathcal{B}_{ijk}^\gamma \sim [Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} - Q_i^{\alpha,a} Q_j^{\gamma,c} Q_k^{\beta,b}] \epsilon_{abc} (C \gamma_5)_{\alpha\beta}, \quad (9)$$

where $a-c$ are color indices, $\alpha-\gamma$ are Dirac indices, and C is the standard charge conjugation matrix. This construction ensures the antisymmetry of the latter two Dirac indices (β, γ), and thus eliminates the flavor-singlet baryon [43].

The pure flavor-singlet baryon field, denoted \mathcal{B}^S , on the other hand, is totally symmetric in quark flavor, Q . This can be encoded into the form of the baryon tensor by permuting the color and Dirac indices while maintaining the flavor-label ordering (ijk) as in Eq. (9), such that the established baryon transformation rules are maintained

$$\begin{aligned} \mathcal{B}_{ijk}^S \gamma \sim & [Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} + Q_i^{\beta,b} Q_j^{\gamma,c} Q_k^{\alpha,a} + Q_i^{\gamma,c} Q_j^{\alpha,a} Q_k^{\beta,b} \\ & + Q_i^{\beta,b} Q_j^{\alpha,a} Q_k^{\gamma,c} + Q_i^{\gamma,c} Q_j^{\beta,b} Q_k^{\alpha,a} + Q_i^{\alpha,a} Q_j^{\gamma,c} Q_k^{\beta,b}] \\ & \times \epsilon_{abc} (C \gamma_5)_{\alpha\beta}. \end{aligned} \quad (10)$$

By transforming the individual quark operators (Q) within the baryon fields of Eqs. (9) and (10), and collecting the transformation matrices (U) together using the commutation rule

[47]

$$Q_i U_{jk} = (-1)^{\eta_i(\eta_j + \eta_k)} U_{jk} Q_i, \quad (11)$$

the resultant transformation rules for the baryon and antibaryons (suppressing the Dirac indices) are

$$\mathcal{B}_{ijk} \rightarrow (-1)^{\eta_{i'}(\eta_j + \eta_{j'}) + (\eta_{i'} + \eta_{j'}) (\eta_k + \eta_{k'})} U_{ii'} U_{jj'} U_{kk'} \mathcal{B}_{i'j'k'}, \quad (12)$$

$$\bar{\mathcal{B}}_{kji} \rightarrow (-1)^{\eta_{i'}(\eta_j + \eta_{j'}) + (\eta_{i'} + \eta_{j'}) (\eta_k + \eta_{k'})} \bar{\mathcal{B}}_{k'j'i'} U_{k'k}^\dagger U_{j'j}^\dagger U_{i'i}^\dagger, \quad (13)$$

for both \mathcal{B} and \mathcal{B}^S .

In matching the elements of the generalized baryon field to those of the standard baryon matrix $B_{kk'}$, the indices are restricted from 1 to 3, and the elements are assigned so that the result is antisymmetric under $i \leftrightarrow j$

$$\mathcal{B}_{ijk} \propto \epsilon_{ijk'} B_{kk'}, \quad (i - k = 1 - 3), \quad (14)$$

where the convention used for the standard baryon field of χ PT is

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix} + \frac{\Lambda'}{2\sqrt{3}} \mathbb{I}_3. \quad (15)$$

Λ' denotes a positive-parity flavor-singlet baryon. Similarly, a matrix B^* can be defined with the same form, representing the lowest-lying negative-parity baryons. The negative-parity flavor-singlet baryon is then denoted by Λ'^* .

The baryon construction of Eq. (14) contains both octet and singlet fields. However, it is known that interactions that include a singlet baryon have couplings that are not related via flavor symmetry to the interaction couplings between octet baryons only. It is therefore important to separate the baryon field into ‘octet-only’ and ‘singlet-only’ pieces, so that the terms in the Lagrangian can be assigned independent coefficients.

To extract the purely octet component of \mathcal{B}_{ijk} , Labrenz and Sharpe antisymmetrized over the j and k indices of Q in Eq. (9), so that the symmetric component (containing the singlet) is removed. In terms of the standard baryon field, with indices i, j , and k restricted to 1–3 and fermion anti-commutation taken into account, this takes the following form

$$\mathcal{B}_{ijk} = \frac{1}{\sqrt{6}} (\epsilon_{ijk'} B_{kk'} + \epsilon_{ikk'} B_{jk'}). \quad (16)$$

The factor of $1/\sqrt{6}$ is chosen so that the baryon wave functions have the correct normalization. This becomes apparent when assigning conventional baryon labels to the elements of the \mathcal{B} tensor (discussed in Sec III). For example, extracting the proton states involves selecting the elements \mathcal{B}_{112} , \mathcal{B}_{121} and \mathcal{B}_{211} , with weights of $1/\sqrt{6}$, $1/\sqrt{6}$ and $-2/\sqrt{6}$, respectively, whose squares sum to 1.

Similarly, the baryon tensor containing the flavor-singlet baryon follows from Eq. (10) for \mathcal{B}^S

$$\mathcal{B}_{ijk}^S = \frac{1}{3\sqrt{2}} \left(+\epsilon_{ijk'} B_{kk'} + \epsilon_{jkk'} B_{ik'} + \epsilon_{kik'} B_{jk'} - \epsilon_{jik'} B_{kk'} - \epsilon_{kjk'} B_{ik'} - \epsilon_{ikk'} B_{jk'} \right). \quad (17)$$

Again, the normalization factor of $(3\sqrt{2})^{-1}$ ensures the squares of the coefficients sum to 1. For the Λ' , one considers the six terms, \mathcal{B}_{123}^S , \mathcal{B}_{132}^S , \mathcal{B}_{213}^S , \mathcal{B}_{231}^S , \mathcal{B}_{312}^S and \mathcal{B}_{321}^S , each of which provides a contribution with magnitude $\Lambda'/\sqrt{6}$. Similarly

$$\epsilon_{ijk} \mathcal{B}_{ijk}^S = \sqrt{6} \Lambda', \quad (i - k = 1 - 3). \quad (18)$$

The symmetry relations that describe the relationships among the elements of the baryon tensors are important when considering their general form in the graded symmetry group, *i.e.* where the indices $i - k$ run from 1 to 6. For the octet field \mathcal{B} , the following two symmetry relations are sufficient to specify the two group invariants of the symmetry, α and β [43], which are related to the standard D and F couplings

$$\mathcal{B}_{ikj} = (-1)^{\eta_j \eta_k + 1} \mathcal{B}_{ijk}, \quad (19a)$$

$$\mathcal{B}_{ijk} = (-1)^{\eta_i \eta_j} \mathcal{B}_{jik} + (-1)^{\eta_i \eta_j + \eta_j \eta_k + \eta_k \eta_i} \mathcal{B}_{kji}. \quad (19b)$$

Similarly, the following symmetry relations exist for the field \mathcal{B}^S

$$\mathcal{B}_{jik}^S = (-1)^{\eta_i \eta_j} \mathcal{B}_{ijk}^S, \quad (20a)$$

$$\mathcal{B}_{ikj}^S = (-1)^{\eta_j \eta_k} \mathcal{B}_{ijk}^S, \quad (20b)$$

$$\mathcal{B}_{jki}^S = (-1)^{\eta_i \eta_k + \eta_i \eta_j} \mathcal{B}_{ijk}^S, \quad (20c)$$

$$\mathcal{B}_{kij}^S = (-1)^{\eta_i \eta_k + \eta_j \eta_k} \mathcal{B}_{ijk}^S, \quad (20d)$$

$$\mathcal{B}_{kji}^S = (-1)^{\eta_i \eta_j + \eta_i \eta_k + \eta_j \eta_k} \mathcal{B}_{ijk}^S. \quad (20e)$$

C. The Lagrangian

The singlet Lagrangian will couple the singlet-baryon field to a combination of octet-meson and octet-baryon fields, $A \cdot \mathcal{B}$. Given the maximal symmetry for \mathcal{B}^S in Eq. (10), there are three different ways of contracting the octet-baryon flavor indices. In keeping with the baryon construction of Labrenz and Sharpe [43], we consider a Lagrangian term similar to their term with coefficient α , $\bar{\mathcal{B}}_{kji}^S A_{kk'} \mathcal{B}_{ijk'}$, where the first two indices of \mathcal{B} are contracted with \mathcal{B}^S ; a term similar to their term with coefficient β , $\bar{\mathcal{B}}_{kji}^S A_{ik'} \mathcal{B}_{k'jk}$, where the latter two indices of \mathcal{B} are contracted with \mathcal{B}^S ; and a third candidate $\bar{\mathcal{B}}_{kji}^S A_{kk'} \mathcal{B}_{ik'j}$, where the outer indices of \mathcal{B} are contracted with \mathcal{B}^S . Noting that the octet field of Eq. (9) has a symmetry constraint under the exchange of the latter two indices as in Eq. (19a), this third candidate is related to the first candidate and may be discarded. Similarly, the relation of Eq. (20a) for \mathcal{B}^S is opposite to that of Eq. (19a) for \mathcal{B} and therefore the second candidate vanishes. Thus, there is only one group invariant combination.

In the general case where all indices run from 1 – 6, the commutation/anticommutation behavior of the elements of the transformation matrices U depends on their location in the block matrix. As a result, grading factors must be included when swapping the order of the indices. Noting that Eq. (20e) indicates that \mathcal{B}_{ijk}^S is antisymmetric under exchange of the outer indices in both the normal quark and single ghost-quark sectors, the interaction Lagrangians take the following forms

$$\mathcal{L}_S = 2\sqrt{2} g_s (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})} \bar{\mathcal{B}}_{kji}^S A_{kk'} \mathcal{B}_{ijk'}, \quad (21)$$

$$\mathcal{L}_{SS} = \sqrt{6} g_{ss} (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})} \bar{\mathcal{B}}_{kji}^S A_{kk'} \mathcal{B}_{ijk}^S. \quad (22)$$

We note these are special cases of the general irreducible representations of graded Lie groups classified by Balantekin and Bars [58, 59]. The leading coefficient in \mathcal{L}_S is selected to define the coupling associated with the process $\Lambda' \rightarrow \pi_0 \Sigma_0$ as g_s .

Our investigation of the separation of quark-flow connected and disconnected contributions to standard baryons constrains the indices $i - k$ to 1 – 3, whereas $k' = 1 - 6$ incorporates the ghost meson-baryon contributions, which reveal the disconnected contributions.

In summary, the first-order interaction Lagrangian for low-lying octet and singlet mesons, octet baryons and low-lying negative-parity singlet baryons, including the Dirac structure, takes the form

$$\begin{aligned} \mathcal{L}_{BA}^{(1)} &= \mathcal{L}_\alpha + \mathcal{L}_\beta + (\mathcal{L}_S + \text{h.c.}) + \mathcal{L}_{SS}, \\ &= 2\alpha (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})} \bar{\mathcal{B}}_{kji} \gamma_\mu \gamma_5 A_{kk'}^\mu \mathcal{B}_{ijk'} \\ &\quad + 2\beta \bar{\mathcal{B}}_{kji} \gamma_\mu \gamma_5 A_{ik'}^\mu \mathcal{B}_{k'jk} \\ &\quad + 2\sqrt{2} g_s (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})} \\ &\quad \times \left\{ \bar{\mathcal{B}}_{kji}^{S*} \gamma_\mu A_{kk'}^\mu \mathcal{B}_{ijk'} + \bar{\mathcal{B}}_{k'ji} \gamma_\mu A_{kk'}^\mu \mathcal{B}_{ijk}^{S*} \right\} \\ &\quad + \sqrt{6} g_{ss} (-1)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})} \bar{\mathcal{B}}_{kji}^{S*} \gamma_\mu \gamma_5 A_{kk'}^\mu \mathcal{B}_{ijk'}^{S*}, \end{aligned} \quad (23)$$

where \mathcal{B}_{ijk}^{S*} denotes the odd-parity generalized field associated with B^* and thus Λ'^* via Eq. (17). The loss of γ_5 in the terms coupling octet- and singlet-flavor baryons reflects the odd parity of the low-lying Λ'^* .

III. GHOST BARYON STATE IDENTIFICATION

In this section we review briefly the process of relating the elements of \mathcal{B} and \mathcal{B}^S to baryons having one ghost quark and two ordinary quarks. This is relevant to our investigation of the ordinary Λ'^* baryon, where one ghost quark can be introduced through the disconnected sea-quark loop. The ordinary baryons (those without ghost quark content) adopt the labeling convention assigned in Eqs. (15) through (17). The case for one ghost quark in \mathcal{B} was first present by Chen and Savage [46]. We will extend the formalism to include \mathcal{B}^S .

In identifying the intermediate states of the meson-baryon channels dressing a baryon, we find it is useful to assign state

labels that provide information about the charge, quark composition and flavor symmetry. Thus, in defining the baryon labels, there are some differences in our notation with respect to Ref. [46]. Here, the choice in baryon labels is to manifest the symmetry and flavor properties of baryons with a single ghost quark. If one of the quarks is a (commuting) ghost particle, it may be combined with the two remaining ordinary quarks in a Σ -like symmetric or a Λ -like antisymmetric configuration of ordinary quarks. For example, for a proton with a ghost quark, one defines new particles $\tilde{\Sigma}_{p,\tilde{u}}^+$ and $\tilde{\Lambda}_{p,\tilde{u}}^+$, which correspond to symmetric and antisymmetric combinations of the two remaining quark flavors, respectively.

In addition, unphysical states such as doubly-charged ‘protons’ $\tilde{\Sigma}_{p,\tilde{u}}^{++}(\tilde{u}, u, u)$ and negatively charged ‘neutrons’ $\tilde{\Sigma}_{n,\tilde{d}}^-(\tilde{d}, d, d)$ are members of the single ghost-quark baryons. These states occur in both the graded-symmetry approach and the diagrammatic approach by considering all possible quark-flow topologies. Note that these states do not contribute in full QCD, where the cancellation of the connected and disconnected quark-flow diagrams associated with them enforces the Pauli exclusion principle.

Following Ref. [46], the flavor-symmetric variants are encoded in a rank-3 tensor s_{ijk} , which is symmetric in the last two indices reserved for ordinary quarks

$$\begin{aligned} s_{411} &= \tilde{\Sigma}_{p,\tilde{u}}^{++}, & s_{511} &= \tilde{\Sigma}_{p,\tilde{d}}^+, & s_{611} &= \tilde{\Sigma}_{\Sigma,\tilde{s}}^+, \\ s_{412} &= \tilde{\Sigma}_{p,\tilde{u}}^+/\sqrt{2}, & s_{512} &= \tilde{\Sigma}_{n,\tilde{d}}^0/\sqrt{2}, & s_{612} &= \tilde{\Sigma}_{\Sigma,\tilde{s}}^0/\sqrt{2}, \\ s_{422} &= \tilde{\Sigma}_{n,\tilde{u}}^0, & s_{522} &= \tilde{\Sigma}_{n,\tilde{d}}^-, & s_{622} &= \tilde{\Sigma}_{\Sigma,\tilde{s}}^-, \\ s_{413} &= \tilde{\Sigma}_{\Sigma,\tilde{u}}^+/\sqrt{2}, & s_{513} &= \tilde{\Sigma}_{\Sigma,\tilde{d}}^0/\sqrt{2}, & s_{613} &= \tilde{\Sigma}_{\Xi,\tilde{s}}^0/\sqrt{2}, \\ s_{423} &= \tilde{\Sigma}_{\Sigma,\tilde{u}}^0/\sqrt{2}, & s_{523} &= \tilde{\Sigma}_{\Sigma,\tilde{d}}^-/\sqrt{2}, & s_{623} &= \tilde{\Sigma}_{\Xi,\tilde{s}}^-/\sqrt{2}, \\ s_{433} &= \tilde{\Sigma}_{\Xi,\tilde{u}}^0, & s_{533} &= \tilde{\Sigma}_{\Xi,\tilde{d}}^-, & s_{633} &= \tilde{\Sigma}_{\Xi,\tilde{s}}^-. \end{aligned} \quad (24)$$

The baryon subscripts reflect the combined number of strange quarks and strange ghost quarks and the superscripts record the total electric charge. The flavor of the residing ghost quark is explicit in the subscript. We note that although Ω has been used to denote the (\tilde{s}, s, s) combination, this baryon is actually associated with octet as opposed to decuplet baryons.

Using a similar notation, the Λ -like particles are encoded in an antisymmetric tensor, denoted t_{ij}

$$\begin{aligned} t_{41} &= \tilde{\Lambda}_{\Sigma,\tilde{u}}^0, & t_{51} &= \tilde{\Lambda}_{\Sigma,\tilde{d}}^-, & t_{61} &= \tilde{\Lambda}_{\Xi,\tilde{s}}^-, \\ t_{42} &= \tilde{\Lambda}_{\Sigma,\tilde{u}}^+, & t_{52} &= \tilde{\Lambda}_{\Sigma,\tilde{d}}^0, & t_{62} &= \tilde{\Lambda}_{\Xi,\tilde{s}}^0, \\ t_{43} &= \tilde{\Lambda}_{p,\tilde{u}}^+, & t_{53} &= \tilde{\Lambda}_{n,\tilde{d}}^0, & t_{63} &= \tilde{\Lambda}_{\Sigma,\tilde{s}}^0. \end{aligned} \quad (25)$$

These baryons are distributed over the elements of \mathcal{B} in the following manner. The manifestly symmetric part of \mathcal{B} has the simple relation

$$\mathcal{B}_{ijk} = \sqrt{\frac{2}{3}} s_{ijk}, \quad (i = 4 - 6, j - k = 1 - 3), \quad (26)$$

which places the ghost quark in the first index position of \mathcal{B} . The symmetric states with ghost quarks mapped to the second

and third index positions of \mathcal{B} are combined with the antisymmetric baryons of Eq. (25) in

$$\mathcal{B}_{jik} = \frac{1}{2} t_{ip} \epsilon_{pjk} + \frac{1}{\sqrt{6}} s_{ijk}, \quad (27a)$$

$$\mathcal{B}_{jki} = -\mathcal{B}_{jik}, \quad (27b)$$

where $i = 4 - 6$, $j - k = 1 - 3$ and $p = 1 - 3$. These two equations map the symmetric baryon states to two elements of \mathcal{B} with weight $1/\sqrt{6}$ such that the weighting of $\sqrt{2/3}$ in Eq. (26) provides a sum of squares equal to one. Similarly, Eqs. (27) distribute the antisymmetric ghost baryon states of Eqs. (25) over four elements of \mathcal{B} with weight $1/2$ such that a sum of squares again equals one.

The complete flavor antisymmetry of \mathcal{B}^S for ordinary quarks restricts its first ghost sector to baryon states having antisymmetry in the ordinary quarks, *i.e.* the baryons of Eq. (25)

$$\mathcal{B}_{ijk}^S = \frac{1}{\sqrt{6}} t'_{ip} \epsilon_{pjk}, \quad (28a)$$

$$\mathcal{B}_{jik}^S = +\mathcal{B}_{ijk}^S, \quad (28b)$$

$$\mathcal{B}_{jki}^S = +\mathcal{B}_{ijk}^S. \quad (28c)$$

where $i = 4 - 6$ and $j - k = 1 - 3$. Eqs. (28), now with a prime to denote the association with the flavor-singlet baryon sector, maps each baryon of Eqs. (25) over six elements of \mathcal{B}^S . Analogous results are obtained for the odd-parity \mathcal{B}_{ijk}^{S*} using t'_{ip} .

In studies of the Λ'^* , the low-lying ghost states reside in the octet meson and octet baryon sectors. The odd-parity ghost states of \mathcal{B}_{ijk}^{S*} are of relevance to other states transiting to Λ'^* and a meson. As an example, consider an octet baryon transition to a singlet baryon and an octet meson. Such physics is relevant to understanding the disconnected loop contributions in $\Sigma \rightarrow \pi \Lambda'^*$ or $\Xi \rightarrow \bar{K} \Lambda'^*$. These become particularly important in lattice QCD, where the initial state can be a finite-volume excitation associated with a resonance.

This defines the method by which baryon labels are assigned to different elements of the three-index baryon tensors, \mathcal{B} and \mathcal{B}^S , and their odd-parity analogues. By writing out the full Lagrangian in Eq. (23), one can simply pick out the relevant contributions to each baryon in each interaction channel available for the symmetry subgroup $SU(3|3)_V$.

IV. FLAVOR-SINGLET Λ -BARYON COUPLINGS

In this section we examine the coupling strengths for transitions involving the flavor-singlet component, Λ'^* . As described at the beginning of Sec. II, the couplings of the ghost sector in $SU(3|3)_V$ provide the couplings of quark-flow disconnected sea-quark loop contributions in $SU(6|3)_V$ that we are seeking. In writing out the relevant terms of the Lagrangian, we report the meson-baryon \rightarrow baryon transition terms, noting that the baryon \rightarrow meson-baryon terms are given by the Hermitian conjugate.

A. Flavor-octet baryon dressings of the singlet baryon

Here we consider the specific case relevant to the low-lying odd-parity $\Lambda(1405)$ baryon, known to have an important flavor-singlet component [4]. We commence by considering the process $\Lambda'^* \rightarrow \mathcal{B} \Phi \rightarrow \Lambda'^*$ via the Lagrangian of Eq. (23).

In this case, the u , d and s valence quarks that comprise the Λ'^* state may each contribute to the Φ meson loop, shown in Fig. 1. The ghost quark sector may enter through the disconnected loop, and therefore there are three choices for the antiquark plus three choices for the ghost antiquark. Noting that the flavor-singlet η' meson cannot be combined with an octet baryon to form a flavor singlet, there are $(3 \times 6) - 1 = 17$ terms in the Lagrangian for the $\bar{\Lambda}'^* \Phi \mathcal{B}$ coupling. At one loop, the baryon \mathcal{B} contains at most one ghost quark for which the meson contains the corresponding antighost quark.

The $SU(3|3)_V$ -flavor composition of the interaction Lagrangian of Eq. (23) contains the regular full-QCD meson-baryon couplings to the singlet Λ'^*

$$\mathcal{L}_{S, \bar{\Lambda}'^*}^{\text{QCD}} = g_s \bar{\Lambda}'^* \left\{ \bar{K}^0 n + K^- p + \pi^+ \Sigma^- + \pi^0 \Sigma^0 + \pi^- \Sigma^+ + K^+ \Xi^- + K^0 \Xi^0 + \eta \Lambda \right\}, \quad (29)$$

and the ghost meson to ghost baryon couplings

$$\mathcal{L}_{S, \bar{\Lambda}'^*}^{\text{Ghost}} = -g_s \bar{\Lambda}'^* \left\{ \sqrt{\frac{2}{3}} \left(\tilde{K}^- \tilde{\Lambda}_{p, \tilde{u}}^+ + \tilde{K}^0 \tilde{\Lambda}_{n, \tilde{d}}^+ + \tilde{\pi}^+ \tilde{\Lambda}_{\Sigma, \tilde{d}}^- + \tilde{\pi}^- \tilde{\Lambda}_{\Sigma, \tilde{u}}^+ + \tilde{K}^+ \tilde{\Lambda}_{\Xi, \tilde{s}}^- + \tilde{K}^0 \tilde{\Lambda}_{\Xi, \tilde{s}}^0 \right) + \frac{1}{\sqrt{3}} \left(\tilde{\pi}_0 \tilde{\Lambda}_{\Sigma, \tilde{u}}^0 + \tilde{\pi}_0 \tilde{\Lambda}_{\Sigma, \tilde{d}}^0 \right) + \frac{1}{3} \left(\tilde{\eta} \tilde{\Lambda}_{\Sigma, \tilde{u}}^0 + \tilde{\eta} \tilde{\Lambda}_{\Sigma, \tilde{d}}^0 - 2 \tilde{\eta} \tilde{\Lambda}_{\Sigma, \tilde{s}}^0 \right) \right\}, \quad (30)$$

where the Dirac structure has been suppressed. In the full-QCD case, the relative coupling strengths of the $\pi \Sigma$, $\bar{K} N$, $K \Xi$ and $\eta \Lambda$ channels are consistent with early work [60] as expected.

The ghost terms, however, present novel features. Only antisymmetric $\tilde{\Lambda}$ ghost baryons contribute. Thus, the coupling of a baryon with a single ghost quark to a flavor-singlet baryon is constrained to have its two ordinary quarks in an antisymmetric formation. The $\tilde{\Sigma}$ baryons do not contribute.

In addition, flavor-singlet symmetry is manifest. Each meson charge state participating in the quark-flow disconnected loop contributes in a uniform manner with a coupling strength of $\frac{2}{3} g_s^2$.

The ghost-quark subscript on the baryon field enables the identification of the flavor of the quark participating in the loop. Thus, matrix elements can be separated into their quark-flow connected and disconnected parts for each quark flavor. Similarly, the baryon subscripts facilitate the incorporation of

$SU(3)$ -flavor breaking effects through variation of the hadron masses.

The essential feature to be drawn from this analysis is that the relative contribution of quark-flow connected and disconnected diagrams is now known. Consider the process $\Lambda'^* \rightarrow K^- p \rightarrow \Lambda'^*$ proportional to g_s^2 . This process proceeds through a combination of quark-flow connected diagrams and the quark-flow disconnected diagram of Fig. 1. See for example, Ref. [51]. The ghost Lagrangian reports a term

$$-g_s \sqrt{\frac{2}{3}} \bar{\Lambda}'^* \tilde{K}^- \tilde{\Lambda}_{p,\tilde{u}}^+, \quad (31)$$

indicating that this process proceeds through the quark-flow disconnected diagram with strength $\frac{2}{3} g_s^2$. That is, two thirds of the full-QCD process $\Lambda'^* \rightarrow K^- p \rightarrow \Lambda'^*$ proceeds by the quark-flow disconnected diagram with a u quark flavor participating in the loop. The fact that two-thirds of the strength lies in the loop is perhaps surprising.

The full-QCD processes involving the neutral π^0 or η mesons are only slightly more complicated, as one needs to sum over more than one quark flavor participating in the sea-

quark loop. After summing over the participating flavors, each full-QCD process proceeds with one third of the strength in quark-flow connected diagrams and two-thirds of the strength in quark-flow disconnected diagrams.

B. Flavor-singlet baryon dressings of octet baryons

It is also of interest to examine processes where the flavor-singlet baryon participates as an intermediate state dressing of an octet baryon. The full Lagrangian describing the processes $\mathcal{B} \rightarrow \Phi \mathcal{B}^{S*} \rightarrow \mathcal{B}$, where \mathcal{B} is an octet baryon, can likewise be decomposed into full-QCD and ghost meson to ghost baryon components, suppressing the Dirac structure

$$\begin{aligned} \mathcal{L}_{S,\Lambda'^*}^{\text{QCD}} = g_s \Big\{ & \bar{n} (K^0 \Lambda'^*) + \bar{p} (K^+ \Lambda'^*) \\ & + \bar{\Sigma}^- (\pi^- \Lambda'^*) + \bar{\Sigma}^0 (\pi^0 \Lambda'^*) + \bar{\Sigma}^+ (\pi^+ \Lambda'^*) \\ & + \bar{\Xi}^- (K^- \Lambda'^*) + \bar{\Xi}^0 (\bar{K}^0 \Lambda'^*) \\ & + \bar{\Lambda} (\eta \Lambda'^*) \Big\}, \end{aligned} \quad (32)$$

$$\begin{aligned} \mathcal{L}_{S,\Lambda'^*}^{\text{Ghost}} = g_s \Big\{ & \bar{n} \left(\tilde{\pi}^- \tilde{\Lambda}_{p,\tilde{u}}^{+'*} + \tilde{\eta}_d \tilde{\Lambda}_{n,\tilde{d}}^{0'*} + \tilde{K}^0 \tilde{\Lambda}_{\Sigma,\tilde{s}}^{0'*} \right) + \bar{p} \left(\tilde{\eta}_u \tilde{\Lambda}_{p,\tilde{u}}^{+'*} + \tilde{\pi}^+ \tilde{\Lambda}_{n,\tilde{d}}^{0'*} + \tilde{K}^+ \tilde{\Lambda}_{\Sigma,\tilde{s}}^{0'*} \right) \\ & + \bar{\Sigma}^- \left(\tilde{\pi}^- \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} + \tilde{\eta}_d \tilde{\Lambda}_{\Sigma,\tilde{d}}^{-' *} + \tilde{K}^0 \tilde{\Lambda}_{\Xi,\tilde{s}}^{-' *} \right) + \bar{\Sigma}^+ \left(\tilde{\eta}_u \tilde{\Lambda}_{\Sigma,\tilde{u}}^{+'*} + \tilde{\pi}^+ \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} + \tilde{K}^+ \tilde{\Lambda}_{\Xi,\tilde{s}}^{0'*} \right) \\ & + \frac{1}{\sqrt{2}} \bar{\Sigma}^0 \left(-\tilde{\pi}^- \tilde{\Lambda}_{\Sigma,\tilde{u}}^{+'*} + \tilde{\eta}_u \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} - \tilde{\eta}_d \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} + \tilde{\pi}^+ \tilde{\Lambda}_{\Sigma,\tilde{d}}^{-' *} - \tilde{K}^0 \tilde{\Lambda}_{\Xi,\tilde{s}}^{0'*} + \tilde{K}^+ \tilde{\Lambda}_{\Xi,\tilde{s}}^{-' *} \right) \\ & + \bar{\Xi}^- \left(\tilde{K}^- \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} + \bar{\tilde{K}}^0 \tilde{\Lambda}_{\Sigma,\tilde{d}}^{-' *} + \tilde{\eta}_s \tilde{\Lambda}_{\Xi,\tilde{s}}^{-' *} \right) + \bar{\Xi}^0 \left(\tilde{K}^- \tilde{\Lambda}_{\Sigma,\tilde{u}}^{+'*} + \bar{\tilde{K}}^0 \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} + \tilde{\eta}_s \tilde{\Lambda}_{\Xi,\tilde{s}}^{0'*} \right) \\ & + \frac{1}{\sqrt{6}} \bar{\Lambda} \left(\tilde{\pi}^- \tilde{\Lambda}_{\Sigma,\tilde{u}}^{+'*} + \tilde{\eta}_u \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} + \tilde{\eta}_d \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} - 2\tilde{\eta}_s \tilde{\Lambda}_{\Sigma,\tilde{s}}^{0'*} + \tilde{\pi}^+ \tilde{\Lambda}_{\Sigma,\tilde{d}}^{-' *} - 2\tilde{K}^- \tilde{\Lambda}_{p,\tilde{u}}^{+'*} - 2\bar{\tilde{K}}^0 \tilde{\Lambda}_{n,\tilde{d}}^{0'*} + \tilde{K}^0 \tilde{\Lambda}_{\Xi,\tilde{s}}^{0'*} + \tilde{K}^+ \tilde{\Lambda}_{\Xi,\tilde{s}}^{-' *} \right) \Big\}. \end{aligned} \quad (33)$$

In each channel of Eq. (33), there are terms which represent the disconnected loop contributions of the normal full-QCD interactions of Eq. (32), and there are additional terms that serve to cancel the unphysical quark-flow connected diagrams associated with each channel. For example, in the case of the neutron channel \bar{n} , the relevant ghost term $\tilde{K}^0 \tilde{\Lambda}_{\Sigma,\tilde{s}}^{0'*}$ comes with the same magnitude as the normal process, g_s , indicating that there is no completely-connected contribution for this process. The physical process proceeds through a quark-flow disconnected diagram. The other two terms in this channel, $\tilde{\pi}^- \tilde{\Lambda}_{p,\tilde{u}}^{+'*}$ and $\tilde{\eta}_d \tilde{\Lambda}_{n,\tilde{d}}^{0'*}$ occur to cancel the aforementioned unphysical connected diagrams. These contributions are also important and must be included when evaluating the quark-flow disconnected contributions to baryon matrix elements.

Note that, unlike the flavor-singlet Λ -baryon couplings of Eq. (30), where the structure of the $\tilde{\pi}^0$ and $\tilde{\eta}$ is manifest, here

there is no $s\bar{s}$ contribution in any channel apart from the octet $\bar{\Lambda}$. In the $\bar{\Sigma}$, where both light flavor η_q mesons appear, the flavor symmetry of the pion is apparent. Similarly, in the $\bar{\Lambda}$ channel, the flavor symmetry of the η_q contributions identifies the octet- η participating to cancel the full-QCD process $\bar{\Lambda} (\eta \Lambda'^*)$. In all other cases, the light-quark η_u and η_d mesons propagate with the mass of the pion.

C. Flavor-singlet baryon dressings of the singlet baryon

Finally, we also consider the quark-flow disconnected loop contributions to the process $\Lambda'^* \rightarrow \Phi \mathcal{B}^{S*} \rightarrow \Lambda'^*$. We find

$$\mathcal{L}_{SS,\Lambda'^*}^{\text{QCD}} = g_{ss} \bar{\Lambda}'^* \eta' \Lambda'^*, \quad (34)$$

and

$$\begin{aligned} \mathcal{L}_{SS,\Lambda'^*}^{\text{Ghost}} = \frac{g_{ss}}{\sqrt{2}} \bar{\Lambda}'^* \Bigg\{ & \sqrt{\frac{2}{3}} \left(\tilde{K}^{-'} \tilde{\Lambda}_{p,\tilde{u}}^{+'*} + \tilde{K}^{-0'} \tilde{\Lambda}_{n,\tilde{d}}^{0'*} \right. \\ & + \tilde{\pi}^{+'} \tilde{\Lambda}_{\Sigma,\tilde{d}}^{-'*} + \tilde{\pi}^{-'} \tilde{\Lambda}_{\Sigma,\tilde{u}}^{+'*} \\ & + \tilde{K}^{+'} \tilde{\Lambda}_{\Xi,\tilde{s}}^{-'*} + \tilde{K}^{0'} \tilde{\Lambda}_{\Xi,\tilde{s}}^{0'*} \Big) \\ & + \frac{1}{\sqrt{3}} \left(\tilde{\pi}^{0'} \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} - \tilde{\pi}^{0'} \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} \right) \\ & \left. + \frac{1}{3} \left(\tilde{\eta}' \tilde{\Lambda}_{\Sigma,\tilde{u}}^{0'*} + \tilde{\eta}' \tilde{\Lambda}_{\Sigma,\tilde{d}}^{0'*} - 2 \tilde{\eta}' \tilde{\Lambda}_{\Sigma,\tilde{s}}^{0'*} \right) \right\}. \end{aligned} \quad (35)$$

Here we have added a prime to the meson labels to remind the reader that in full QCD these contributions enter with the meson propagating with the mass of the flavor-singlet η' . The symbols serve to indicate the manner in which specific quark-flow disconnected loops contribute to the full-QCD process.

Again, we observe the physical process proceeding through a quark-flow disconnected diagram. Here, the final term, indicating the ghost- η' contributions, provides a total contribution with strength g_{ss} , precisely removing the full-QCD contribution of Eq. (34). The remaining ghost terms enter to ensure unphysical contributions from quark-flow connected diagrams do not contribute in full QCD. Their contributions are important and must be included when evaluating the quark-flow disconnected contributions to baryon matrix elements.

V. CONCLUSION

An extension of the graded symmetry approach to partially quenched QCD has been presented. The extension builds on previous theory by explicitly including flavor-singlet baryons in its construction. The aim is to determine the strength of both the quark-flow connected and quark-flow disconnected diagrams encountered in full-QCD calculations of hadronic matrix elements containing flavor-singlet baryon components.

This is particularly important in light of recent progress in the calculation of the electromagnetic properties of the $\Lambda(1405)$ in lattice QCD [13], where flavor-singlet symmetry

is known to play an important role [4]. There only the quark-flow connected contributions are calculated.

Because a determination of the quark-flow disconnected contributions to baryon excited states presents formidable challenges to the lattice QCD community, it is essential to have an understanding of the physics missing in these contemporary lattice QCD simulations. An understanding of the couplings of the quark-flow disconnected contributions to baryon matrix elements is vital to understanding QCD and ultimately comparing with experiment.

We have discovered that the full-QCD processes

$$\begin{aligned} \Lambda'^* &\rightarrow K^- p \rightarrow \Lambda'^*, & \Lambda'^* &\rightarrow \bar{K}^0 n \rightarrow \Lambda'^*, \\ \Lambda'^* &\rightarrow \pi^+ \Sigma^- \rightarrow \Lambda'^*, & \Lambda'^* &\rightarrow \pi^0 \Sigma^0 \rightarrow \Lambda'^*, \\ \Lambda'^* &\rightarrow \pi^- \Sigma^+ \rightarrow \Lambda'^*, & \Lambda'^* &\rightarrow K^+ \Xi^- \rightarrow \Lambda'^*, \\ \Lambda'^* &\rightarrow K^0 \Xi^0 \rightarrow \Lambda'^* \quad \text{and} \quad \Lambda'^* &\rightarrow \eta \Lambda \rightarrow \Lambda'^*, \end{aligned}$$

proceed with two thirds of the full-QCD strength in the quark-flow disconnected diagram. Details of the quark flavors participating in the loops and their relative contributions are provided in Eq. (30) of Sec. IV A.

We have also considered the separation of quark-flow connected and disconnected contributions for the flavor-singlet baryon dressings of octet baryons in Sec. IV B, and flavor-singlet baryon dressings of the the flavor-singlet baryon in Sec. IV C.

When combined with previous results for flavor-octet baryons, this information enables a complete understanding of the contributions of disconnected sea-quark loops in the matrix elements of baryons and their excitations, including the $\Lambda(1405)$. This allows an estimation of quark-flow disconnected corrections to lattice QCD matrix elements, or an adjustment of other observables prior to making a comparison with the quark-flow connected results of lattice QCD.

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